Compression of electroencephalograms using tensor decompositions

Justin Dauwels*, Srinivasan Kannan‡*, Ramasubba Reddy M.‡, and Andrzej Cichocki†,
* Nanyang Technological University, Singapore
E-mail: justin@dauwels.com
‡ Indian Institute of Technology Madras, Chennai, India
E-mail: srinivasan.sivam@gmail.com
E-mail: rsreddy@iitm.ac.in
† RIKEN BSI, Laboratory for Advanced Brain Signal Processing, Wako-Shi, Japan
E-mail: cia@brain.riken.jp

Abstract

We develop compression schemes for EEG signals based on tensor decomposition. We explore various ways to arrange EEG signals into tensors, and we evaluate several tensor decomposition schemes, including the Tucker decomposition, PARAFAC, and recent random fiber selection approaches. We compute rate-distortion curves for our proposed tensor-based schemes for EEG compression. In addition, we investigate how compression affects common EEG statistics, including the power spectrum and statistical dependence measures ("EEG synchrony").

I. Introduction

Electroencephalograms (EEG) are electrical signals recorded along the scalp or brain surface, generated by firing of neurons within the brain [1], [2]; those brain signals are instrumental in several areas. In neurology, EEG signals are for example used to diagnose and study epilepsy and sleep disorders. In that context, EEG signals are often recorded over a short period of time, typically 20–40 minutes. However, EEG signals are sometimes continuously recorded over extended periods of time (several days, weeks, or potentially even months):

- in neurology intensive care units (ICU), e.g., for stroke patients,
- in telemedicine for neurological patients, where EEG is continuously recorded outside the hospital, usually at home.

In recent years, EEG signals are commonly used in Brain-Computer Interfaces (BCI). The latter use brain waves to control devices such as a wheelchair, computer mouse and/or keyboard. BCI systems may provide a communications channel for the motion-disabled. At last, but not at least, EEG signals are intensively studied by neuroscientists, who try to gain insight in how the brain works. Neuroscientists often record EEG signals while subjects are stimulated in a controlled fashion (e.g., visual stimulation), or perform certain well-defined tasks (e.g., memory task).

In telemedicine, intensive care, and other contexts, it is desirable and natural to record scalp EEG over extended periods of time; this results, however, in massive EEG data sets. Therefore, it is meaningful (and often necessary) to compress the EEG signals before storage or transmission. As an illustration, let us consider a night-long recording of EEG from a patient with sleep disorder. Assuming the EEG is acquired from 21 channels at a rate of several hundreds of samples per second with a resolution of 16 bits/sample, the EEG recordings would amount to several GBs. Ideally, one would want to monitor patients over several days, weeks, or perhaps months–especially in the setting of telemedicine; clearly, it is then desirable to compress the EEG in order to limit the storage space and bandwidth to reasonable size.

The main challenges for EEG compression are as follows:

- the number of EEG channels can be large (e.g., 256), especially if accurate inverse modeling is needed,
- high sampling rate may be required (several kHz in the case of cortical EEG; several hundred Hz for scalp EEG), to capture spikes and high-frequency oscillations in the EEG
- high-quality (lossless or near lossless) reconstruction of the EEG is often needed [3]. However, since EEG is highly stochastic and non-stationary, lossless or near lossless EEG compression suffers from low compression performance.
- For telemedicine applications, compression algorithms should operate in real-time while consuming as little power as possible.

In this study, we utilize tensor decomposition methods to compress EEG signals. We explore various ways to arrange EEG signals into tensors, and we evaluate several tensor decomposition schemes [4], [5], including the Tucker decomposition, PARAFAC, and recent random fiber selection approaches (e.g., [6]). We compute rate-distortion curves for our proposed tensor-based schemes for EEG compression. In addition, we investigate how compression affects common EEG statistics, including the spectrum and statistical dependence measures ("EEG synchrony"). Such statistics are commonly used in neurology and in BCI systems.

In this abstract, we present our preliminary results. More detailed results will be presented at the workshop. This abstract is organized as follows.

II. Review of literature

Signals can be compressed by exploiting correlations in those signals. Multiple signals, such as EEG recordings, can be *jointly* compressed by utilizing the correlation between those signals, besides the correlation within each signal *individually*. A variety of techniques have been developed for compressing EEG signals; we refer to [7] for an excellent review on lossless EEG compression.

A. Auto-regressive models

EEG signals are often modeled as auto-regressive (AR) processes, similarly as speech signals. An AR predictor is a non-recursive system that predicts the current signal sample by using a weighted sum of a pre-specified number of previous samples. Perfect reconstruction is made possible by transmitting the prediction error or residuals. Various AR predictors have been developed: linear AR predictor, recursive least squares and adaptive neural network predictors. Further refinements include context-based bias cancellation [3] and detailed prediction residual modeling [8], which improve compression performance at the expense of computational complexity.

B. Transformations

Before compressing signals, it can be fruitful to first transform them in an other domain. In particular, one aims to find a compact representation of the signals. For example, the wavelet transform is commonly used, since it often leads to sparse time-frequency representations. If many of the wavelet coefficients are close to zero, one can set them to zero, and store only the most significant non-zero coefficients; this leads to a compressed representation, at the expense of minor residual errors. Common transformations include discrete cosine transform [7], subband transform, wavelet transform, wavelet-packet transform, and integer lifting wavelet transform [9], [7].

C. From single-channel to multi-channel compression

The methods discussed above are single-channel compression schemes, and they remove only the intra-channel correlation. Multichannel compression algorithms compress all EEG channels simultaneously, by exploiting both inter-channel and intra-channel correlation. Various approaches are possible: for example, one can extend univariate AR prediction to multivariate AR prediction, although such approach does not yet seem to have been followed for EEG compression. In contrast, the integer Karhunen-Loève transform and its sub-optimal variation [10] have been used.

D. Compressed sensing

The emerging field of compressed sensing opens the way to acquire sparse signals with very few random measurements, well below the Nyquist rate. Signals could be acquired directly in the compressed form and reconstructed for subsequent analysis; sparse acquisition of EEG signals is quite attractive for telemedicine. Compressed sensing of EEG has been explored in a few recent studies [11], [12].

E. Discussion

A large number of studies have been devoted to the compression of EEG. However, most of them consider the compression of single EEG channels. On the other hand, multi-way analysis has been applied intensively to EEG signals (especially non-negative decompositions of time-frequency maps), mostly for extracting features (see, e.g., [13], [14]); those studies seem to suggest that multi-way analysis may be effective for EEG compression as well, however, many issues still need to be systematically explored. We are currently conducting such systematic study, and in the following, we will present our preliminary results; we will also outline ongoing research.

III. Tensor Decompositions for Compressing EEG

Tensors or multi-way arrays provide a natural representation for multi-dimensional data. Tensor decomposition models are important tools for feature extraction and classification since they capture the dependencies in higher-order data-sets. They have found application in many areas, e.g., psychometrics, chemometrics, and signal processing [4], [5]. Here, we explore tensor decomposition schemes for compressing multi-channel EEG signals.

A. Tensor formation

Multichannel EEG signals are arranged in the form of a three-way tensor. Let $\mathcal{X} \in \mathbb{R}^{I \times J \times K}$, represent the tensor. So far, we have explored to methods to form a tensor from multi-channel EEG signals:

- **Time tensor** : Single-channel EEG signals are arranged to form a matrix [9]. This matrix is then stacked to form a three-way tensor $\mathcal{X} \in \mathbb{R}^{N \times N \times M}$, whose frontal slices represents signal from a particular channel. It can be expressed by the following relationship:
$$\mathbf{X}_{::k} = \text{matrix}(EEG^{(k)}(1:N^2)) \qquad \forall\, k = 1, \ldots, M, \tag{1}$$
where $N \times N$ is the dimension of the frontal slice, M the number of channels, $EEG^{(k)}$ the EEG from channel k and matrix() is function that arranges EEG in the form of a matrix; the entries are filled starting at the top left-hand side, from left to right on the odd rows, and from right to left on the even rows.
- **Wavelet tensor** : Wavelet tensor is derived from the time tensor $\mathcal{X}$, by subjecting each frontal slice to 2-D discrete wavelet transform. The relationship between the wavelet-tensor $\mathcal{X}^w \in \mathbb{R}^{N \times N \times M}$ and time tensor $\mathcal{X}$ is given by:
$$\mathbf{X}^w_{::k} = \text{2D-DWT}(\mathbf{X}_{::k}) \qquad \forall\, k = 1, \ldots, M. \tag{2}$$
The wavelet decomposition improves the sparsity of the frontal slices, which may lead to more effective tensor decompositions.

In both tensor constructions, the mode-1 (column) fibres represents the samples from the same channel displaced by N-samples. Mode-2 (row) fibres represents the adjacent samples from the same channel, whereas the samples along the mode-3 fibres (tubes) represents the samples from adjacent channel at the same time instant. One can expect strong correlations between the tensor entries along mode-2 and mode-3 fibres, but less so along mode-1 fibres, due to the non-stationary nature of EEG.

B. Tensor decomposition and compression

In this abstract, we only consider the Tucker decomposition: the tensor $\mathcal{X}$ is decomposed into a core tensor $\mathcal{G} \in \mathbf{R}^{P\times Q\times R}$ and factor matrices $\mathbf{A} \in \mathbb{R}^{N\times P}$, $\mathbf{B} \in \mathbb{R}^{N\times Q}$ and $\mathbf{C} \in \mathbb{R}^{M\times R}$. (We refer to [4], [5] for a review of tensor decompositions. We use the same notation as in [4]). The factor matrices are *basis matrices* capturing the variation along the three modes and the core tensor captures the interaction between them. The Tucker decomposition is expressed mathematically by:

$$\mathcal{X} = \mathcal{G} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C} + \mathcal{E} = \tilde{\mathcal{X}} + \mathcal{E} \tag{3}$$

where $\tilde{\mathcal{X}}$ is the Tucker decomposition of $\mathcal{X}$ and $\mathcal{E}$ represents the approximation error.

C. Compression Performance measures

The compression performance is measured using two parameters: compression ratio and percent-root mean square difference (PRD). The compression ratio is defined as the ratio of number of entries in the original tensor and the number of entries in the compressed tensor:

$$CR = \frac{\texttt{numel}(\mathcal{X})}{\texttt{numel}(\mathcal{G}) + \texttt{numel}(\mathbf{A}) + \texttt{numel}(\mathbf{B}) + \texttt{numel}(\mathbf{C})}, \tag{4}$$

where $\texttt{numel}()$ is the number of entries in a matrix or tensor. Note that for tensor constructions, the number of entries in the original tensor $\texttt{numel}(\mathcal{X})$ is equal to the size of the EEG data set, i.e., number of channels times the length of the EEG signals.

The other important performance measure is the percent-root mean square difference (PRD) between the original and the reconstructed signal:

$$PRD(\%) = \sqrt{\frac{\sum_i^{N^2}(x(i) - \tilde{x}(i))^2}{\sum_i^{N^2} x(i)^2}} \times 100. \tag{5}$$

The latter quantifies the distortion between the original tensor $\mathcal{X}$ and the Tucker decomposition $\tilde{\mathcal{X}}$.

IV. Results

We have analyzed 64-channel EEG from Physiobank EEG-MMI database [15]. The EEG was recorded at a sampling rate of 80Hz. We considered EEG segments that are 256 and 1024 samples long, and the size of the tensor $\mathcal{X}$ is chosen as $16 \times 16 \times 64$ and $32 \times 32 \times 64$ respectively. We first computed the Tucker decomposition for all possible sizes of the core tensor $\mathcal{G}$. Next, for fixed compression ratios (CR) we chose the core tensor size that minimizes the distortion (PRD). As a benchmark, we have applied singular value decomposition (SVD) as well: the EEG signals are organized in a matrix where each row contain the EEG signal of one channel; SVD is applied to that matrix, and the number of retained singular vectors is varied from 1 to M, yielding different compression ratios and corresponding PRDs. Our preliminary results are summarized in Fig. 1. We have obtained those results by 4-fold crossvalidation. Indeed, since the problem of selecting the size of the core tensor is essentially a model selection problem, crossvalidation is necessary to assess the generalization performance. The tensor-based compression schemes clearly outperform SVD, especially at large compression ratios. The tensor construction does not seem to be matter much: time tensor and wavelet tensors lead to about the same results. Interestingly, the compression performance is the best for the long EEG segments (1024 samples); that is most clearly the case for SVD, and much less pronounced for the tensor-based compression schemes. The latter is good news: it implies that good compression performance can be achieved even with fairly short EEG segments; as a result, the computational complexity of the tensor decomposition can be kept small, and hence, the multi-way EEG compression scheme has low complexity. Examples of reconstructed EEG signals are shown in Fig. 2.

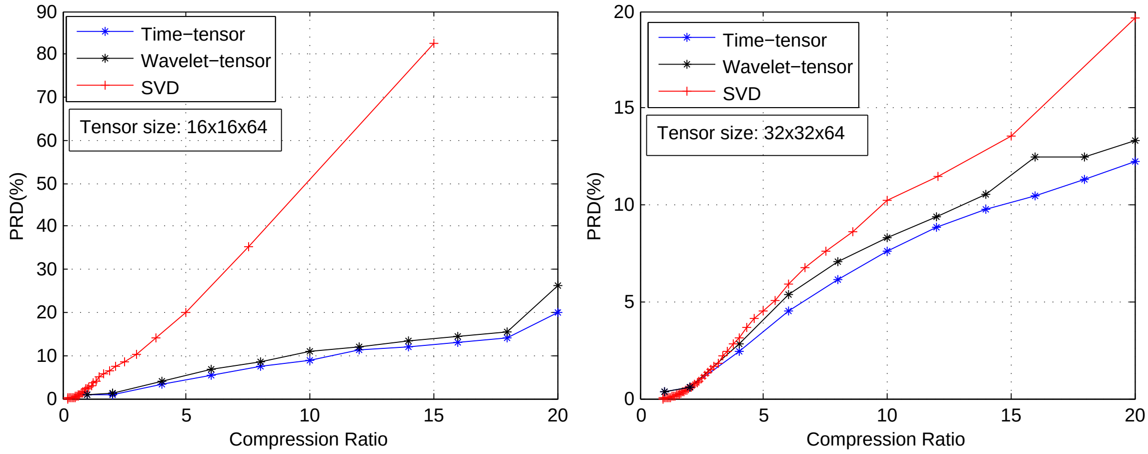
We have also computed the relative error on specific EEG statistics, including power spectrum and correlation matrix. Similar trends as in Fig. 1 can be observed (not shown here).

V. Discussion and conclusion

Conventional multi-channel compression schemes usually exploit the intra-channel and inter-channel correlation in separate stages; tensor-based compression schemes capture intra-channel and inter-channel correlations simultaneously, which is a more elegant approach that seems to yield good compression performance. In the near future, we will compare the proposed method additional multi-channel compression schemes, besides SVD.

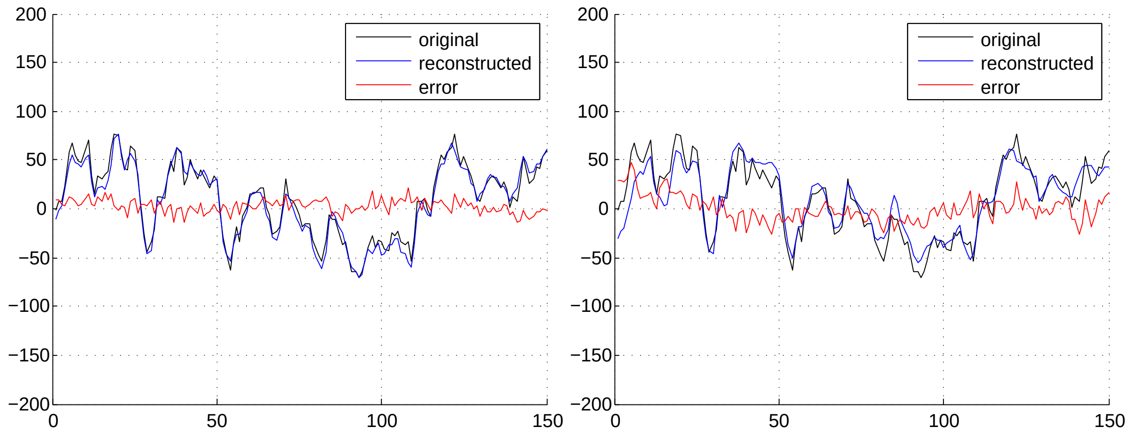
We are currently exploring extensions of the proposed approach. For example, the error tensor $\mathcal{E}$ may be further decomposed by another Tucker decomposition. Alternatively, other compression techniques may be applied to the cores tensor and factor matrices, e.g., arithmetic coding. Such additional compression stages try to exploit remaining correlations in the core tensor and factor matrices, and/or error tensor.

So far, we have only obtained results for Tucker decomposition. We are now experimenting with other tensor models, including PARAFAC [4], [5] and approximations based on partial sampling [6]. We are also considering unequal error protection; it is well-known that the high frequency components in EEG signals are the most contaminated, since they are usually the weakest. Therefore, we can allow higher approximation error for the entries in the wavelet tensor corresponding to high-frequency EEG



(a) (b)

Fig. 1. Comparison of EEG compression scheme based on SVD and Tucker decomposition. In SVD the number of singular vectors is increased to achieve the desired distortion; in Tucker decomposition the size of core tensor is optimized. Results are shown for tensor $\mathcal{X}$ of size $16 \times 16 \times 64$ (left) and $32 \times 32 \times 64$ (right).



(a) PRD=5%, CR=5.65 (b) PRD=10%, CR=14.83

Fig. 2. Original and reconstructed EEG signal, and error signal for the time tensor model for PRD = 5% (left) and 10% (right).

components. At the same time, we would like to keep the approximation errors for the low-frequency entries as small as possible. At last, we are at present analyzing several additional datasets, from healthy subjects and from epilepsy and AD patients. Results from our ongoing research will be presented at the workshop.

REFERENCES

[1] P. Nunez and R. Srinivasan, *Electric Fields of the Brain: The Neurophysics of EEG*, Oxford University Press, 2006.

[2] S. Sanei and J. Chambers, *EEG signal processing*, Wiley-Interscience, 2007.

[3] N. Memon, X. Kong, and J. Cinkler, "Context-based lossless and near-lossless compression of EEG signals," *IEEE Transactions on Biomedical Engineering*, vol. 3, pp. 231–238, mar. 1999.

[4] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," *SIAM Review*, vol. 51, no. 3, pp. 455–500, September 2009.

[5] A. Cichocki, R. Zdunek, Anh Huy Phan, and S. Amari, *Nonnegative Matrix and Tensor Factorizations: Applications to Exploratory Multi-way Data Analysis and Blind Source Separation*, Wiley, 2009.

[6] C. Caiafa and A. Cichocki, "Generalizing the columnrow matrix decomposition to multi-way arrays", Linear Algebra and Applications, 433 (2010) 557-573.

[7] G. Antoniol and P. Tonella, "EEG data compression techniques," *IEEE Transactions on Biomedical engineering,*, vol. 44, pp. 105–114, Feb. 1997.

[8] N.Sriraam and C.Eswaran, "An adaptive error modeling scheme for the lossless compression of EEG signals," *IEEE Transactions on Information Technology in Biomedicine*, vol. 12, pp. 587–594, Sep. 2008.

[9] K.Srinivasan and M. R. Reddy, "Efficient preprocessing technique for lossless real-time EEG compression," *Electronics Letters*, vol. 46, pp. 26–27, Jan. 2010.

[10] Y. Wongsawat, *On the study of multi-channel EEG: Lossless compression, signal modeling and classification*. PhD thesis, Graduate School of The University of Texas at Arlington, Dec. 2007.

[11] S. Aviyente, "Compressed Sensing Framework for EEG Compression," *Proceedings of IEEE Statistical Signal Processing Workshop*, August 2007.

[12] K.-K. Poh and P. Marziliano, "Compressive sampling of EEG signals with finite rate of innovation," *EURASIP Journal on Advances in Signal Processing*, vol. 2010, pp. 1–12, 2010.

[13] M. Mørup, L. K. Hansen, C. S. Herrmann, J. Parnas, and S. M. Arnfred. "Parallel factor analysis as an exploratory tool for wavelet transformed event-related EEG." NeuroImage, 29:938–947, 2006.

[14] A. Cichocki, Y. Washizawa, T. Rutkowski, H. Bakardjian, A.-H. Phan, S. Choi, H. Lee, Q. Zhao, L. Zhang, and Y. Li, "Noninvasive BCIs: Multiway signal-processing array decompositions", Computer, vol. 41, no. 10, pp. 34–42, 2008.

[15] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. C. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C.-K. Peng, and H. E. Stanley, "PhysioBank, PhysioToolkit, and PhysioNet : Components of a New Research Resource for Complex Physiologic Signals," *Circulation*, vol. 101, no. 23, pp. e215–220, 2000.